CALIFORNIA INSTITUTE OF TECHNOLOGY

EARTHQUAKE RESEARCH LABORATORY

A Dislocation Theory of Earthquakes

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A DISLOCATION THEORY OF EARTHQUAKES

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ABSTRACT

A mechanism for the generation of seismic waves is postulated that is based on the release of shear strain dislocations. A certain probability of release of dislocations is also postulated. From these there are deduced expressions for the frequency distribution of shocks, energy released, etc., that are in agreement with observations. It is shown that a random superposition of pulses such as those released by individual dislocations will form an accelerogram that has the appearance and properties of recorded accelerograms. The relation between the maximum ground acceleration and the size of slip area is examined. It is concluded that the existing recorded strong ground motion accelerograms are reliable samples of possible strong ground motions.

INTRODUCTION

The usual type of California earthquake is associated with a relative horizontal slipping of the two faces of a fault which lies essentially in a vertical plane. As a consequence of this slipping, the surface of the ground experiences a severe motion in the neighborhood of the fault during large earthquakes. This surface ground motion can be measured, and for the relatively small number of strong ground motions that have been recorded the maximum horizontal acceleration measured was 0.33 of gravity. For obvious reasons the accumulation of data on strong ground motion is a slow process, so that it will be many years before a sufficiently large body of observations is amassed to give a reasonably complete picture of what can be expected in the way of destructive seismic motions. Analysis of the problem is rendered difficult by the fact that it is not possible to make direct observations of the mechanism that generates the seismic waves, for the usual California earthquake originates at a depth of approximately 10 miles, although in some large earthquakes the slipping on the fault has extended to the surface of the ground. Although direct observations cannot be made, a mechanism for the generation of the seismic waves can be deduced. This, of course, must satisfy certain conditions. It must be physically self-consistent, and the surface phenomena derived from it must agree with observations of actual earthquakes. If it does this the theory may be used with some confidence to investigate items that have not been measured as yet. In particular it can be used to study what can be expected over a large number of earthquakes. In view of the fact that there is incomplete knowledge of the physical properties of the earth's crust at depths of 10 miles, it is not feasible to attempt to analyze the detailed behavior of the subsurface phenomena; however, assuming average properties and introducing simplifications it is possible to investigate average behavior.

The essence of the present theory is a postulated mechanism of slipping along a fault that will account for the strong motions observed

at the surface of the ground. The mechanism of slipping can be pictured as a releasing of shear dislocations. An example of a shear dislocation, as defined in this paper, is shown in Figures 1a and 1b. Consider an indefinitely extended elastic material, in the interior of which there is a plane crack with boundary 'a' as shown in Figure 1a. The two abutting faces of the crack have been given a lateral displacement relative to each other as shown in Figure 1b. In order to maintain this relative displacement there must exist equal and opposite shear stresses on the two faces. These shear stresses can exist only if friction forces can be developed and this requires that compressive stresses exist in the material.
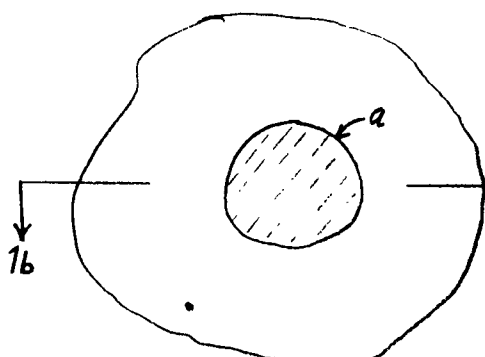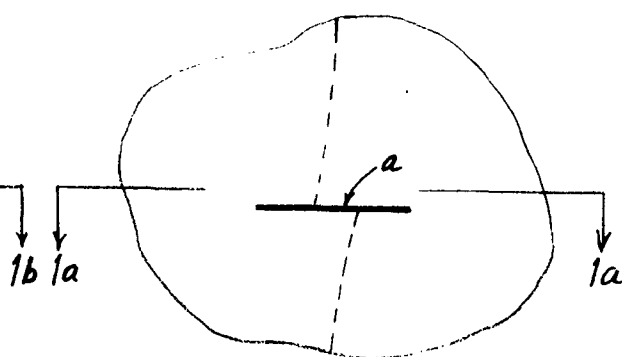

Figure 1a   Figure 1b

In a typical dislocation the shear stresses on the faces of the dislocation and in the adjacent material will be distributed in the manner shown in Figures 2a and 2b. The precise form of the shear distribution will depend upon the relative displacement of the two faces and the shape of the dislocation. If the stressed dislocation suddenly snaps to the unstressed position stress waves will be propagated out from the dislocation.

If a single shear dislocation were located in the earth's crust and it were to release its stress suddenly it would generate an elemental earthquake. Such elemental shocks have been recorded and in appearance the seismogram is similar to one cycle of a sine wave followed by a small oscillatory tail, as sketched in Figure 3. The pulse released by a dislocation must be of this form for the displacement, velocity and acceleration of a point on the surface of the ground must return to
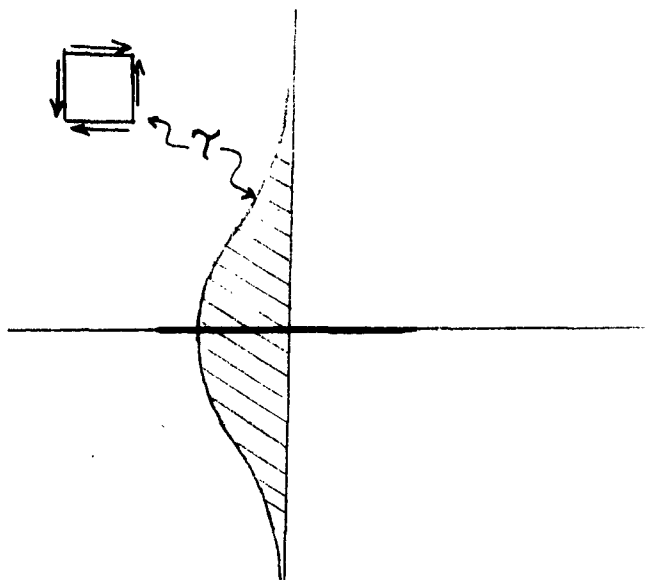
Figure 2a

Stress along a line perpendicular
to the dislocation.
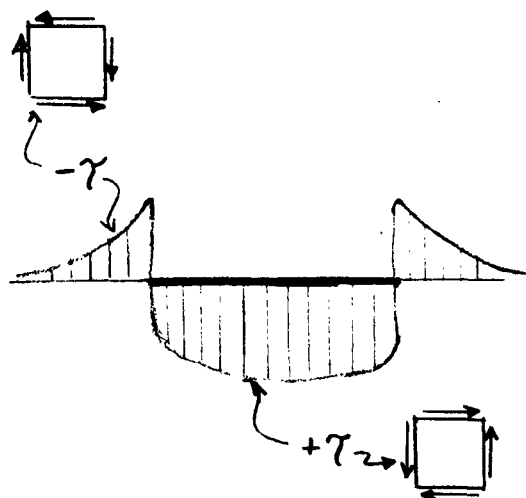
Figure 2b

Stress along a line in the plane
of the dislocation.

zero after the pulse has passed. The reversed shear stresses outside of
the boundary of the dislocation, as shown in Figure 2b, are responsible
for the second loop of the pulse.

Figure 3

If a cluster of dislocations were spread over a surface, as indi-
cated in Figure 4, there would be formed a large crack with boundary A,
which may be called a composite dislocation. The shear stresses on the
faces of the composite dislocation, A, will vary over the area, depend-
ing upon the stresses of the incremental dislocations of which it is

4.

Figure 4

formed and upon the manner of superposition. The stresses associated
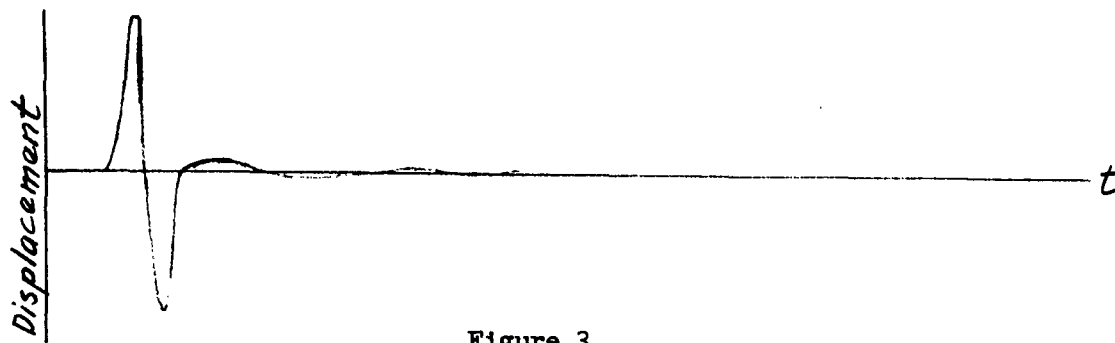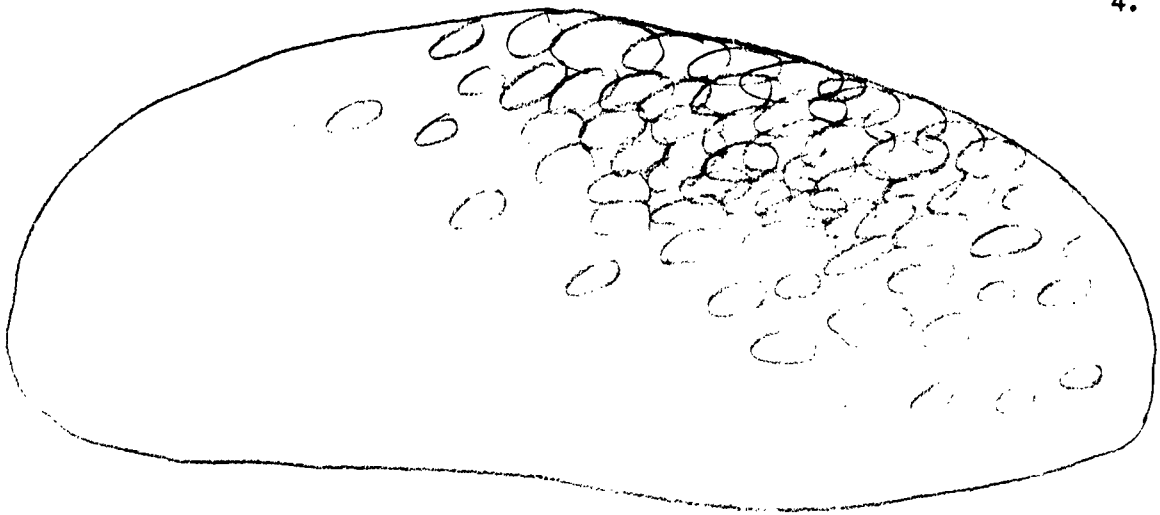with a shear dislocation in an indefinitely extended solid are self-
equilibrating so that an earthquake fault may be considered to be
formed by a planar distribution of shear dislocations, that is, these
form the crack, and superposed is a state of shear produced by relative
horizontal translation of separated points of the earth's crust. The
superimposed shear strain is relieved by the release of the disloca-
tions. Their action may be thought of as being similar in nature to
the so-called "friction chattering" sometimes observed when the plane
faces of two bodies are rubbed together.

The shear stresses on the faces of a stressed dislocation are
in opposite directions on the two faces so that the shear stresses are
equal and opposite with respect to the plane of the dislocation. This
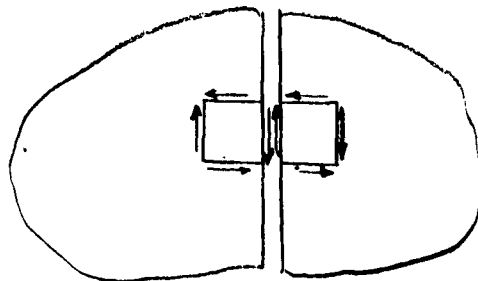
Antisymmetrical State of Stress

antisymmetrical state of stress is shown in the diagram. When the dis-
location is suddenly released, the stress waves generated will be anti-

symmetrical with respect to the plane of the dislocation. This means that points on the plane of the dislocation will not undergo displacement during the passage of the stress waves. As a consequence of this it can be expected that during an earthquake the surface ground motion will be less intense along the plane of the fault than at relatively short distances from it. If the earth's crust were homogeneous and isotropic there would be no motion on the plane of the dislocation. Geological inhomogeneities, wave reflections, etc., will tend to mask the effect but it can be expected that the surface intensity along the fault will be less than would be obtained if the shear waves were symmetrical with respect to the fault. Customary methods of extrapolating surface intensities of ground motions assume that the disturbances originate in a symmetrical fashion and it has happened that these methods indicate greater intensities along the fault near the epicenter than are actually observed.

The stresses and strains associated with a shear dislocation may be computed in the following manner. The stress distribution produced by a force acting at a point in an indefinitely extended solid is[1]

$$\sigma_r = \frac{P}{8\pi(1-\nu)}\left\{ (1-2\nu)z(r^2+z^2)^{-3/2} - 3r^2 z(r^2+z^2)^{-5/2} \right\}$$

$$\sigma_\theta = \frac{P}{8\pi(1-\nu)}\left\{ (1-2\nu)z(r^2+z^2)^{-3/2} \right\}$$

$$\sigma_z = \frac{P}{8\pi(1-\nu)}\left\{ z(r^2+z^2)^{-3/2} + 3z^3(r^2+z^2)^{-5/2} \right\}$$

$$\tau_{rz} = -\frac{P}{8\pi(1-\nu)}\left\{ (1-2\nu)r(r^2+z^2)^{-3/2} + 3rz^2(r^2+z^2)^{-5/2} \right\}$$
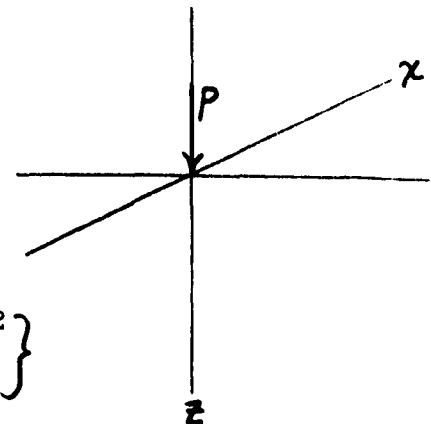
where P is the force acting at the origin of the coordinate system in the z-direction, $x = r\cos\theta$, $y = r\sin\theta$, and ν is Poissons ratio for the material. σ_r, σ_θ and σ_z are the normal stresses in the r, θ, z directions, and τ_{rz} is the rz shear stress, $\tau_{r\theta}$ and $\tau_{z\theta}$ being zero. The

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[1] S. Timoshenko, Theory of Elasticity, McGraw-Hill Book Co., New York, 1934, pg. 321.

partial derivative of the stresses with respect to x gives the stress distribution associated with a doublet of forces as shown in the accompanying diagram. Each force of this doublet is the resultant shear on the face of an infinitesimal dislocation of area dydz lying in the y-z plane at the origin. For example, the shear stress γ_{rz} associated with the doublet is

$$\gamma_{rz} = \frac{-Px}{8\pi(1-\nu)} \left\{ (1-2\nu) \frac{z^2-2x^2-2y^2}{(x^2+y^2)^{1/2}(x^2+y^2+z^2)^{5/2}} + \frac{3z^2(z^2-4x^2-4y^2)}{(x^2+y^2)^{1/2}(x^2+y^2+z^2)^{7/2}} \right\}$$

Along the x-axis, perpendicular to the plane of the dislocation, γ_{rz} is equal to γ_{xz} which is given by

$$\gamma_{xz} = \frac{(1-2\nu)}{4\pi(1-\nu)} \frac{1}{x^3}$$

so that the shear stress decreases inversely as the cube of the distance from the origin. The stresses associated with a dislocation of finite size are obtained by integrating the expressions for the infinitesimal dislocation. Each of the stresses for the infinitesimal dislocation is a function of x, y, z of the form P f(x,y,z) . The appropriate integration is

$$\iint pf(x,y-a,z-a)\,da\,db$$

where p is the intensity of shear stress on the face of the dislocation at the point a,b, where a is measured in the x-direction and b is measured in the z-direction. The integration is to extend over the area of the dislocation. Thus the stresses are prescribed over the faces of the dislocation and at a distance from the dislocation the stresses will diminish inversely as the cube of the distance.
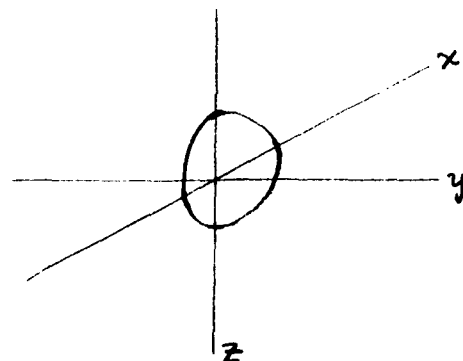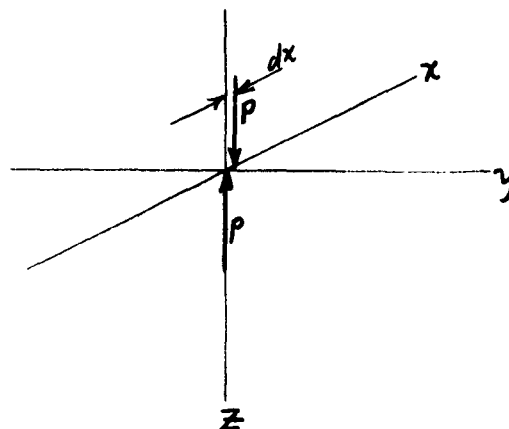
RELEASE OF DISLOCATIONS DURING AN EARTHQUAKE

If an earthquake fault is considered to be formed by a distribution of a large number of dislocations with a superposed state of shear, then as the stresses are built up in the earth's crust eventually the stress at one of the dislocations will exceed the limiting stress and that dislocation will snap. This will reduce the shear between the two faces of that dislocation, but as can be seen from Figure 2b the release of a dislocation will increase the shear stress on the adjacent dislocations. This increase in stress may be sufficient to release an adjacent dislocation. If a second dislocation snaps it in turn will increase the stress on adjacent dislocations and thus trigger a third, etc. This forms a chain reaction which may progress to cover a large area, A, of relative slippage. The process is indicated schematically in a simplified form in Figure 5. The dislocation numbered 1 is the first to be released, and this releases those marked 2. When these second dislocations are released they increase the stress over the area formerly occupied by number 1 and this area snaps a second time. Similarly, when those marked 3 are released they increase the stress and cause those marked 2 to snap a second time, which in turn causes 1 to snap a third time. Actually, the dislocations will not snap in such an orderly manner as here described but can be expected to snap in a rapid, darting, random fashion, but the net result, that is, the total relative slipping, will be essentially as described. The process of slipping along a fault may thus be pictured as rapid releases of dislocations over a spreading area with repeated releases occurring over the area until a position of equilibrium is reached. The stress waves released by a dislocation will have relatively high velocities of propagation but the actual velocity attained by a particle on the face of a dislocation will be small.
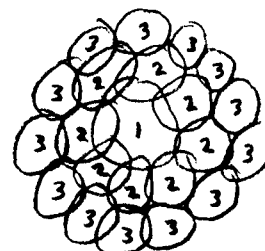

Figure 5

THE MECHANISM OF RELEASE

To determine precisely when and where the individual dislocations are released during an earthquake would require a precise knowledge of the state of stress along the fault, the properties of the material, etc. This is clearly impossible to achieve, however, the problem may be approached from a statistical point of view. That is, a probability mechanism can be postulated that, although not describing precisely the release of each individual dislocation, will describe the average properties of ensembles of dislocations. Again, there is no way of deriving a precise specification of the statistical problem, but this must be deduced from observations of actual earthquakes, a knowledge of the properties of rock, a knowledge of stress distributions associated with shear dislocations of specified type, etc. From such considerations the following mechanism is postulated.

First, the average relative slip between the faces of a composite dislocation of area A is taken to be proportional to the square root of A. This statement is to be understood in a probability sense. That is, it is not required that for every composite dislocation the average slip is precisely proportional to the square root of the area of slip, but it is only

required that the deviations from this rule may be viewed as statistical deviations about a mean and the statements refer to the relation between the mean slip of all earthquakes of a specified area and that area.

The statement that the average slip is proportional to the square root of the slip area implies that the areas are geometrically similar. Actually this can be true only for slip areas up to a certain size, for the usual strong-motion California earthquakes originate at a 10 mile depth and the extent of the slip area is bounded by the surface of the ground. The growth of the slip area is also inhibited because at increasing depths the physical properties of the rock approach those of a material which flows plastically under applied stresses, and therefore cannot maintain shear stresses of as great a magnitude as the material nearer to the surface, although it undoubtedly can rupture in a brittle manner under suddenly applied shear strains. It is presumed that slip areas of increasing size will follow the geometry indicated in the
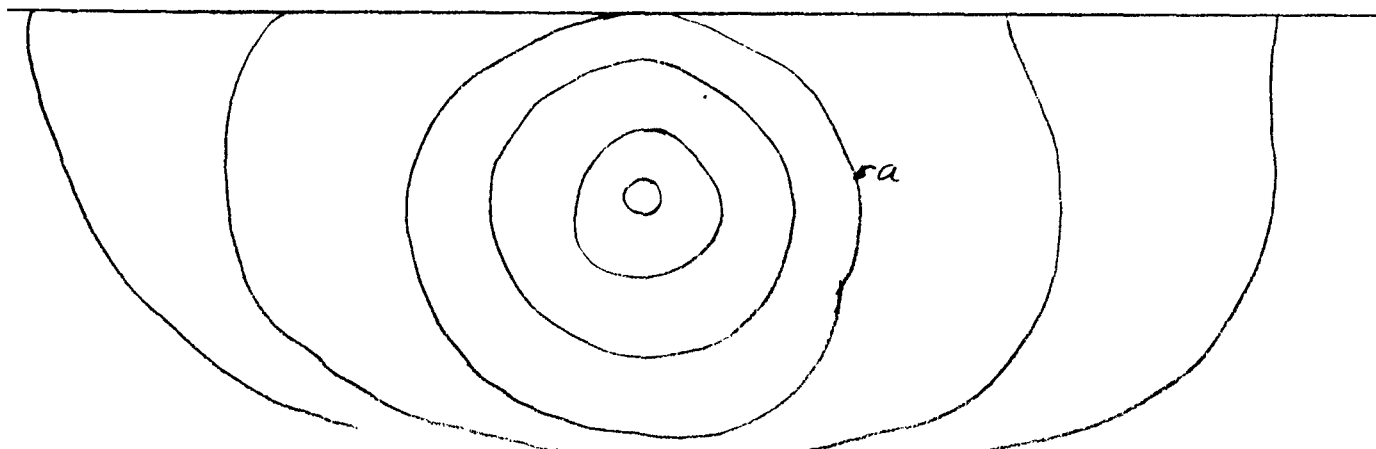
diagram in which all slip areas smaller than 'a' are geometrically similar.
For areas sufficiently larger than 'a' it appears likely that the average
slip will be essentially proportional to the length instead of the square
root of the area. The larger slip areas are of such infrequent occurrence
that little data is available. Because of this, the following discussion
treats only slip areas of the first type and the remarks should properly
be modified where they pertain to the very large earthquakes.

Second, there is required a formulation of the probability of re-
lease of dislocations. The mechanism that controls the size of the slip
area is associated with the distribution of static shear stress along the
fault. It is logical to assume that the distribution of shear stress
varies quite irregularly over the fault plane, that is, something like a
two dimensional, random, continuous function. The variation of stress
along a line lying in the fault plane would have an appearance similar to
the curve shown in the diagram below. As the state of shear strain builds
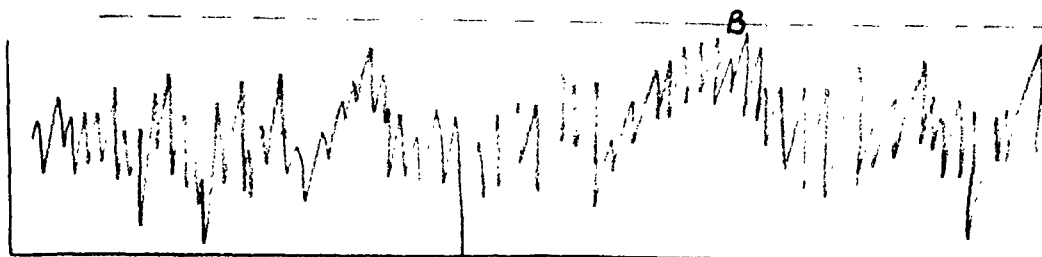
up the stresses increase until one of the peaks reaches the failing point.
This region will then slip and the extent of the slip will be governed by
the region of low stress surrounding the peak. When a region such as B
reaches the failing point a relatively large area of slip will result.

The state of stress over a rectangular area of fault with dimensions ℓ_1 by ℓ_2 can be expressed by

$$\gamma = \sum\sum C_{mn} \sin \frac{n\pi}{\ell_1} x \sin \frac{m\pi}{\ell_2} y$$

and as the state of strain builds up the coefficients C_{mn} increase until a slip occurs at which time there is a sudden change in the coefficients. The relative displacement of the two faces of the fault from the unstressed configuration will also be irregular, but will be much smoother than the stress distribution, and will have an appearance similar to the following diagram. The relative displacement can also be expressed in a Fourier form

$$\mu = \sum\sum B_{mn} \sin \frac{n\pi}{1} x \sin \frac{m\pi}{2} y$$
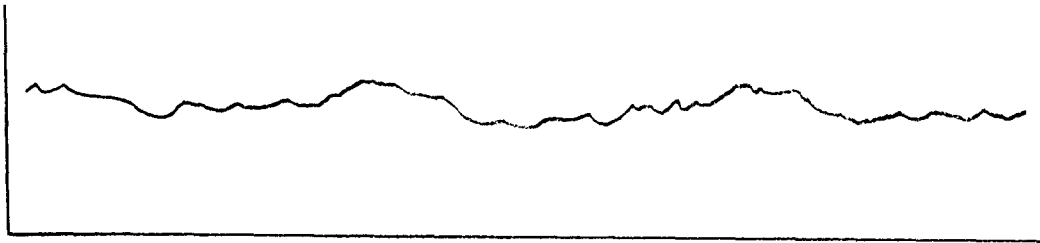
It seems reasonable to assume that the B_{mn} occur in such a way that the frequency of slips of various areas is inversely proportional to the area. That is, the expected number of slips having areas lying between A and A+dA is proportional to $\frac{dA}{A}$. As will be seen, such a frequency distribution for the areas agrees well with observations. It is possible, in principle, to compute the detailed behavior of the coefficients C_{mn} and B_{mn} required for such a frequency distribution and this would give information on the probable distribution of stress and strain along a fault.

Let the frequency distribution of slip areas A, be written

$$f = C \frac{a_o}{A} = C \frac{1}{x}$$

where a_o is the lower limit of A. The total probability must equal unity, so that

$$\int_{x_o}^{x_1} \frac{C}{x} dx = C \log \frac{x_1}{x_o} = 1$$

$$C = \frac{1}{\log \frac{x_1}{x_0}} \frac{1}{x}$$

where x_1 is the maximum possible value of x. The frequency distribution of x is therefore

$$f = \frac{1}{\log \frac{x_1}{x_0}} \frac{1}{x} \tag{1}$$

It may be noted that there must be both an upper limit and a lower limit for x, otherwise $\log \frac{x_1}{x_0}$ becomes infinite and the frequency distribution does not exist.

The mechanism postulated above does not describe the details of a specific earthquake but it describes an average process that should agree with the averages of observations. The mechanism is also a simplification in that it does not include the effects of the variation of physical parameters in the earth's crust. In particular, the fact that with increasing depth the physical properties vary from those of an essentially linearly elastic solid to those of a material that deforms plastically under slowly applied strains, is not included. This means that certain relaxation processes that are undoubtedly operative are not considered. Also, the mechanism excludes from consideration such shocks as may arise from processes other than horizontal slipping along essentially vertical fault planes.

MAGNITUDE AND FREQUENCY

From the viewpoint of the dislocation theory a natural measure of an earthquake is the average slip occurring on the fault. Accordingly, a logarithmic measure of the average slip is taken to define an earthquake, that is, the measure M is related to the average slip by

$$dM = \frac{d\bar{S}}{\bar{S}}$$

where $\bar{S}$ is the average relative slip. In integral form this is

$$\bar{S} = \bar{S}_0 e^M \tag{2}$$

Since the average slip is proportional to the square root of the slip area A it follows that

$$A = A_o \, e^{2M}$$

The frequency distribution of earthquakes given by equation (1) states that the relative frequencies are inversely proportional to the area A so the frequency distribution of shocks in terms of M is

$$f = ce^{-2M} \tag{3}$$

Equation (3) may be compared with recorded data on frequency of occurrence of earthquakes. Such data have been presented by Gutenberg and Richter[2] who give the observed frequencies of shallow focus world earthquakes, Southern California earthquakes and New Zealand earthquakes as

$$\log_{10} N = a + b \, (8 - M) \tag{3a}$$

where N is the mean annual number per tenth magnitude, M is the magnitude of a shock as defined by Richter, and the coefficient b is 0.9, 0.88, 0.87 respectively. These values were obtained by plotting log N against M and fitting a straight line to the data. The points fitted the line closely except for magnitudes greater than 8 in which range the observed frequencies fell below the line with an apparent upper limit for M of 8.7. If the preceeding equation is put in exponential form there is obtained for world earthquakes, Southern California earthquakes and New Zealand earthquakes, respectively,

$$N = c_1 \, e^{-2.07M}$$

$$N = c_2 \, e^{-2.02M}$$

$$N = c_3 \, e^{-2.00M}$$

It is seen that equation (3) fits the data within the limits of observational accuracy.

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[2] B. Gutenberg and C. F. Richter, Seismicity of the Earth, Princeton University Press, 1949.

It is also seen from the foregoing that the measure of an earth-
quake as defined in this paper may be identified with the magnitude de-
fined by Richter. He defined the magnitude as being the logarithm of the
maximum trace amplitude recorded by a certain type of seismograph located
100 kilometers from the epicenter of an earthquake. It will be shown
later that the maximum trace amplitude is proportional to the square root
of the area of slip so that the two definitions are the same.

ENERGY RELEASED

The strain energy stored in a solid by a single shear dislocation
is equal to the work that must be expended by the shear stresses acting
on the faces of the dislocation in producing the relative displacement of
the faces. The strain energy stored by a dislocation of area a_n is thus

$$E_n = \frac{1}{2} \int \tau_n \, S_n \, da_n$$

where τ_n is the shear stress acting on the face and S_n is the relative
displacement (variable) of the two faces. The failing stress τ is the
same for all dislocations, so if $\bar{S}_n$ and $\bar{a}_n$ are the average slip and average
area the total energy released by N dislocations is

$$E = \sum_{n=1}^{N} E_n = \sum c_n \tau \, \bar{S}_n \, \bar{a}_n = \bar{C}N$$

The total energy released is thus proportional to the number N of
dislocations released. The number N is proportional to the area of slip
multiplied by the average slip, over A, so

$$E = C_1 \, A \, \bar{S}$$

or

$$E = C \, e^{3M} \tag{4}$$

This relates the energy released to the magnitude of the shock.

Equation (4) applies only to geometrically similar slip areas.
For example, a small slip area will have longitudinal and vertical dimensions
approximately equal and increasingly larger slip areas will be geometrically

similar to this up to a certain point. The vertical dimensions of slip areas are limited in that a slip originating at a 10 mile depth can extend upward only 10 miles, and because of the changing physical properties of the earth's crust with depth, the slip is limited in the downward direction also. This means that the slip areas of very large earthquakes increase principally by an elongation of the area and that the vertical dimension of the slip area does not increase proportionally. Such slip areas are not geometrically similar to the smaller slip areas and this must be taken into account when computing the energy released. For geometrically similar slip curves the average slip is proportional to the length of the fault. For the smaller shocks this varies as

$$\bar{S} = C' \sqrt{A}$$

However, for very large geometrically similar shocks where the slip area receives no restraint from material above or below, the length is proportional to the area, so that

$$\bar{S} = C'' A$$

In this case the total energy must be written

$$E = C\, e^{4M} \tag{4a}$$

There is, of course, a transition region between the limits of applicability of equations (4) and (4a), and the latter presumably applies only to very large shocks.

The foregoing expressions for energy released may be compared with those derived by an alternate method by Gutenberg and Richter[3]. Consider that at the epicenter the radiated energy arrives in a sinusoidal wave train with maximum acceleration a_o and duration t_o, that is, consider the actual accelerogram to be replaced by an equivalent sinusoidal wave train. Assuming the total energy released to be proportional to the energy reaching the epicenter will then give

$$E = C\, t_o\, a_o^2$$

where a_o is the maximum acceleration at the epicenter. From the recorded

[3] B. Gutenberg and C. F. Richter, Earthquake Magnitude, Energy, Intensity and Acceleration, Bull. Seism. Soc. Amer., Vol. 32, No.3, 1942.

ground accelerations it is found that the following empirical expression relates a_o with M

$$M = 2.2 + 1.8 \log_{10} a_o$$

or

$$a_o = C_1 e^{1.28M}$$

As shown below, the duration may be taken to be

$$t_o = C_2 e^{\frac{M}{2}}$$

These values give for the energy released

$$E = C e^{3.06M} \tag{4c}$$

which agrees with equation (4).

The foregoing analysis does not include the energy carried in the long period components that are found in very large shocks. To allow for this Gutenberg and Richter apply the correction factor e^M thus obtaining

$$E = C e^{4.06M} \tag{4d}$$

which agrees with equation (4b).

AREAS OF SLIP

According to the dislocation theory the total area of relative slip along a fault is proportional to the square of the maximum slip. In terms of the magnitude the area of slip is thus

$$A = A_o e^{2M} \tag{7}$$

This expression should be understood in a probability sense, namely, that on the average shocks of magnitude M will have an area A corresponding to equation (7). As was seen when discussing equation (1) there must be an upper limit for the area A and also a lower limit. The lower limit is the area of the smallest individual shear dislocation that can be released under the conditions applying to an earthquake fault; the upper limit is imposed by the fact that earthquake faults are of finite extent.

The areas corresponding to different values of M can be compared by means of equation (7), for example

$$\frac{A_1}{A_2} = e^{2(M_1 - M_2)} \qquad (8)$$

To investigate the implications of this equation let the shock of El Centro, 18 May 1940 be considered a typical 6.7 magnitude earthquake. Judging from the visible surface slip it is estimated that this shock had a slip area of approximately 40 x 20 = 800 square miles. Using this as a base, the areas of shocks of other magnitudes are given by

$$A = 800 \; e^{2(M-6.7)}$$

This gives for a shock of magnitude M = 0, an area of slip,

$$A_o = 0.0012 \; \text{sq. miles}$$

which is equivalent to a circular area of 210 feet diameter. As will be seen later, from an analysis of strong motion records it appears that an area of this order of magnitude is the lower limit in area of slippage. The relative slip associated with an area of 210 feet diameter may be estimated as follows. The average slip varies with magnitude according to equation (2). For geometrically similar shocks the maximum relative slip is given by the same expression, that is,

$$S = S_o \; e^M$$

The maximum surface slip of the El Centro shock was approximately 15 feet and if the maximum sub-surface slip for a 6.7 magnitude that is the upper bound of a range of geometrically similar shocks is taken to be the same, the slip for shocks of smaller magnitudes is given by

$$S = 15 \; e^{(M-6.7)} \qquad (9)$$

For M = 0 this gives a maximum slip of 0.25 inches. This indicates that the typical smallest dislocation is one of area corresponding to approximately 210 feet in diameter with a maximum relative slip between faces of 0.25 inches.

If the preceding equations are applied to a shock of 8.2 magnitude, such as the 1906 San Francisco earthquake, there is obtained a total area of slip equal to 300 x 55 miles. The surface slip of the 1906

shock disappeared into the ocean north of San Francisco but it appears that the total slip area was of the order of magnitude of the above mentioned figure.

The total movement in California can be estimated by means of the foregoing equations. If the mean annual frequency distribution of shocks in California is taken to be the same per unit area as for Southern California[2] there is obtained

$$f = 0.00086 \ e^{2(8.7-M)}$$

The area of slip per shock according to the preceding calculation is:

$$A = 0.0012 \ e^{2M} \text{ sq. miles}$$

The average relative slip over the area is approximated by one-half the maximum slip, or

$$\overline{S} = \frac{1}{2} (0.25) \ e^{M} \text{ inches.}$$

The total mean annual slipping is given by the integral of $(f A \overline{S})$ and if this is assumed to be distributed uniformly over faults 30 miles deep and 700 miles long there is obtained for the mean annual relative shearing motion of the east and west boundaries of the state

$$\int_{0}^{8.7} \frac{(0.00086)(0.0012)\frac{1}{2}(0.25) \ e^{17.6} \ e^{M}}{(30)(700)} \ dM = 2.2 \text{ inches per year}$$

This may be compared with the estimate of a mean annual relative motion of approximately 2 inches per year that is based on triangulation surveys.

The average duration of the strong motion at the epicenter can be estimated from the motion of a point along the fault. Since such an element of material undergoes a displacement it must be subjected to a force that accelerates and decelerates. If the motion is caused by successive releases of dislocations the force is similar for earthquakes of various magnitudes, differing only in duration. Thus the motion of the point can be written

$$\frac{d^2S}{dt^2} = F(kt)$$

$$\frac{dS}{dt} = \int_0^T F(kt)dt = \frac{1}{k} \int_0^{\Theta_1} F(\Theta)d\Theta \qquad (kt = \Theta)$$

$$S = \frac{1}{k^2} \int_0^{\Theta_1} \left(\int_0^\Theta F(\Theta)d\Theta \right) d\Theta = \frac{C'}{k^2}$$

$$S = \frac{C}{\Theta_1^2} T^2$$

from which

$$T = C_1 \sqrt{S} = C_2 e^{M/2}$$

where S is the maximum slip. The duration estimated from seismograms by Gutenberg and Richter[3] is

$$\log_{10} t_0 = at^{M/4}$$

or

$$t_0 = Ce^{M/1.93}$$

The foregoing values of slip areas, etc., are not exact but are only approximations that are used to show that the formulas give reasonable values when applied to the limits of their ranges and that they are consistent with observations.

ACCELEROGRAMS DERIVED FROM DISLOCATIONS

When the dislocations are released along a fault area they radiate stress pulses and it must be shown that these pulses can form accelerations that agree with those recorded at the surface of the ground. To show this it will be assumed that the release of dislocations sends out elemental shocks that record as one cycle of a sine wave. It is not essential that the pulse be exactly one cycle of a sine wave; this is only a computational convenience. However, it is essential that the pulse be double looped for it will be shown that it is not possible to form a typical accelerogram with single looped pulses such as one-half cycle of a sine wave. It is also necessary that the swarm of pulses have various

wave lengths for it is not possible to form an accelerogram with the
properties of recorded accelerograms with a swarm of pulses all of the
same wave length. As will be seen, the pulses required are predomin-
antly of 2/10 second wave length and the shortest wave length appears
to be something of the order of 1/30 of a second. At a propagation
velocity of 10,000 feet per second a time of 1/30 second corresponds
to a distance of 330 feet so that the size of the smallest dislocation
appears to be of this order of magnitude.

It is assumed that during an earthquake the dislocations are
released over the slip area at random times so that the motion recorded
by an accelerometer is composed of a swarm of pulses random in time. To
determine the required distribution of pulse wave lengths and amplitudes
the following method of analysis is used. A recorded accelerogram is
considered to be a random continuous function and an earthquake is con-
sidered to be a random sample from a parent population. The character-
istics of a random continuous function (accelerogram) are exhibited by
its energy spectrum. The spectra have been computed for recorded strong-
motion accelerograms and the average of these spectra is taken to be
the spectrum of the parent population of strong-motion earthquakes.
The distribution of pulse wave lengths, amplitudes and numbers is de-
termined so that the spectrum of the parent population of pulses is the
same as the above-mentioned average recorded earthquake spectrum.

Consider a swarm of pulses, every one of which is represented
by the acceleration

$$f(t, \theta) = \sin k(t-\theta) \qquad (\theta - \tfrac{\pi}{k} < t < \theta + \tfrac{\pi}{k})$$

where the times of occurrence θ are randomly distributed over an inter-
val of time. If a certain number n, of such pulses are superimposed at
random they will form an accelerogram whose energy spectrum is given by

$$F(k, \nu) = \left(\int \sum_n f(t,k,\theta_n) \sin \nu t \ dt \right)^2 + \left(\int \sum_n f(t,k,\theta_n) \cos \nu t \ dt \right)^2$$

This will, in general, be an irregular curve, but the average of a large
number of spectra from different sets of n pulses will approach a smooth
curve that characterizes the population from which the pulses were taken.

Period $T = 0.2/r$

$$\frac{\cos \frac{\pi}{r}}{1 - r^2}$$

$$\frac{\sin \frac{\pi}{r}}{1 - r^2}$$
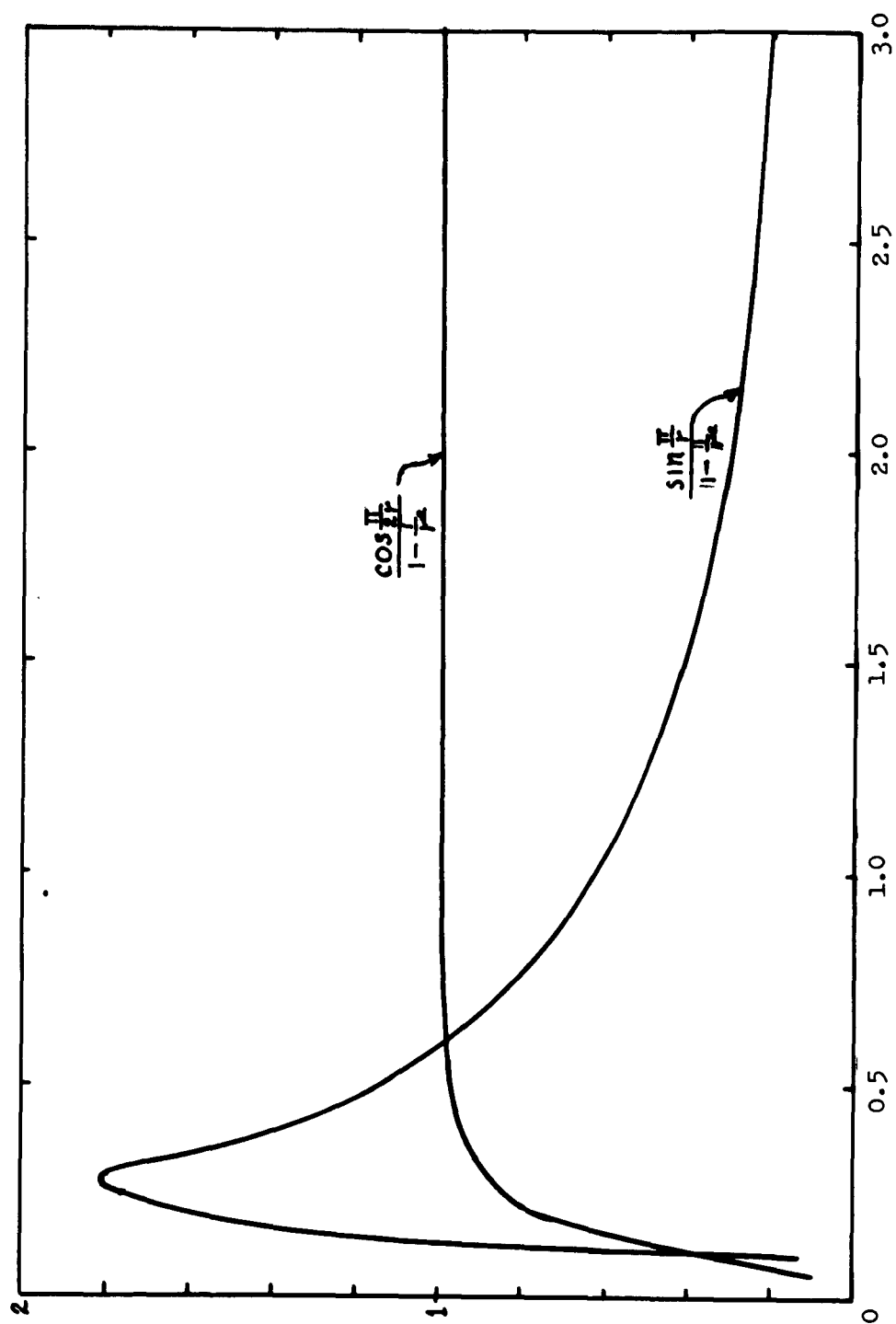
Fig. 6. Spectra for Pulses

To compute the average spectrum it is only necessary to integrate the foregoing expression with respect to Θ, since the probability of Θ is constant with respect to time. When this is done there is obtained for the average spectrum

$$F(k,\nu) = 2\,\frac{\eta}{k^2}\left(\frac{\sin \pi \frac{\nu}{k}}{1-\left(\frac{\nu}{k}\right)^2}\right)^2$$

or

$$F(k,\nu) = 2\,\frac{\eta}{k^2}\left(\frac{\sin \frac{\pi}{r}}{1-\frac{1}{r^2}}\right)^2 \tag{10}$$

where r is the ratio of the wave length being considered on the spectrum to the wave length of the pulses. This is shown in Figure 6, where the square root of the spectrum is drawn. In the case of one-half sine pulses, that is, single loop pulses, the average spectrum is

$$F(k,\nu) = \frac{2\eta}{k^2}\left(\frac{\cos \frac{\pi}{2r}}{1-\frac{1}{r^2}}\right)^2 \tag{11}$$

The square root of this curve is drawn also in Figure 6. These curves are typical in that the double looped pulses produce a humped curve, whereas the single loop pulses do not.

If an accelerogram is formed by the random superposition of pulses of a variety of wave lengths and amplitudes, the mathematical expectation of the spectrum is

$$F(k,\nu) = 2\sum_{k}\left(\frac{\eta_k^{1/2}\,A_k}{k}\quad\frac{\sin \frac{\pi}{r}}{1-\frac{1}{r^2}}\right)^2 \tag{12}$$

where η_k dk is the number of pulses, in the population, having wave lengths lying between k and k + dk, and A_k is the amplitude of the pulse.

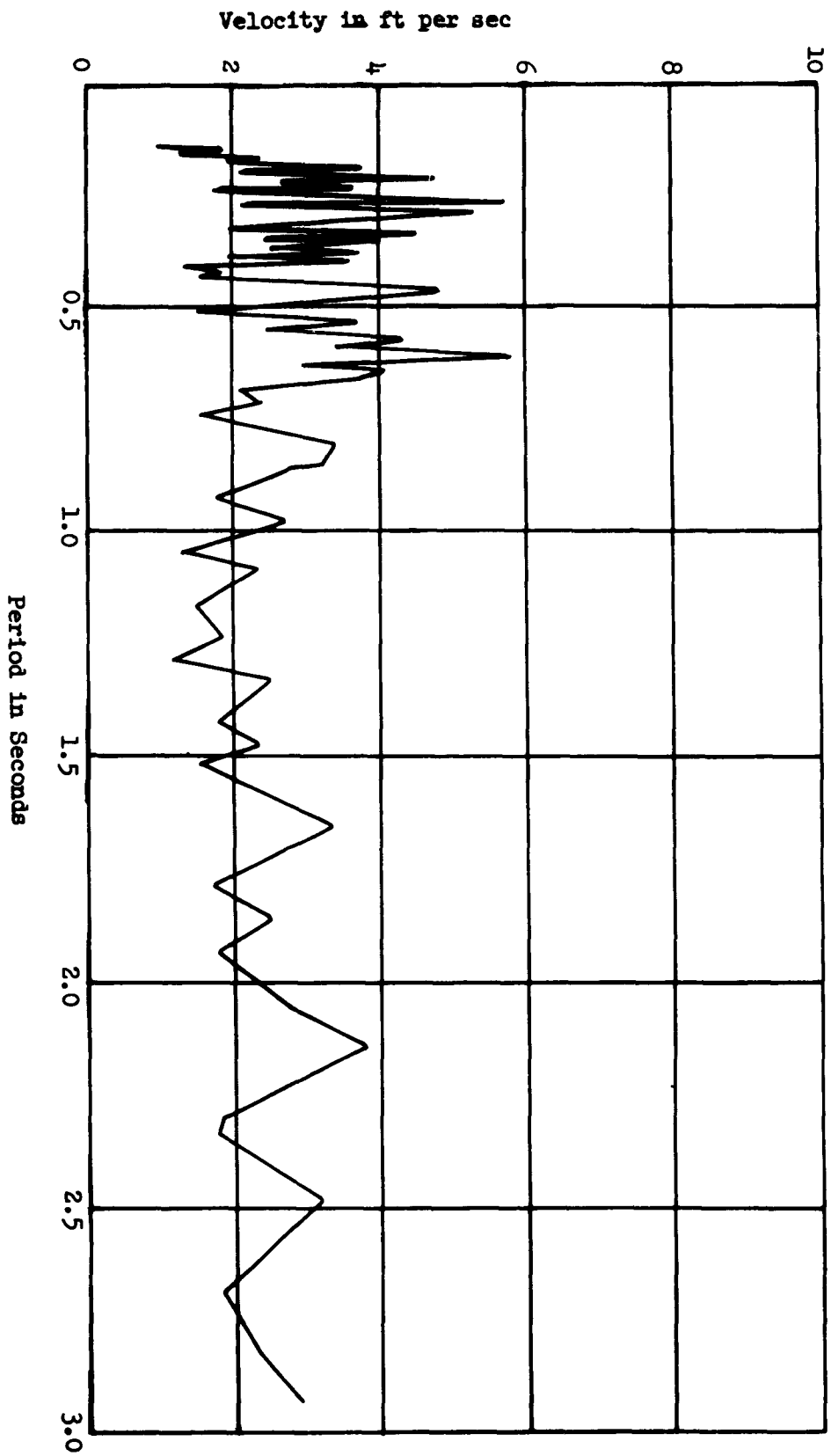A typical earthquake spectrum is shown in Figure 7. The spectrum

Velocity in ft per sec

Period in Seconds

Fig. 7. Spectrum for Olympia, Washington;
Earthquake of April 13, 1949. Component S 80 W.

shown is actually the square root of the energy spectrum[4]. When the computed curves for the two components of the El Centro 18 May 1940, El Centro 30 December 1934, and Olympia, Washington 13 April 1949 strong motion accelerograms are averaged there is obtained a fairly smooth curve that is closely approximated by the curve shown in Figure 8. The averaging in this case was preceded by changing scales so that all of the individual spectra had the same average ordinate. The curve of Figure 8 is taken to be the square root of the average energy spectrum. It is seen that the average spectrum has a hump and therefore cannot be derived from single loop pulses. It can, however, be derived from double loop pulses. When equation (12) is fitted to the curve of Figure 8 the values of $\left(\dfrac{n_k^{1/2} A_k}{k}\right)$ are determined,

and these are shown in Figure 9. Since the numbers $n_k^{1/2}$ and the amplitudes A_k occur as a product it is impossible to distinguish their separate effects on the spectrum, for quadrupling the number has the same effect as doubling the amplitude of the pulses. If, for example, the relative numbers are taken to be inversely proportional to the square of their wave lengths that is, $n_k = k^2$, the curve of Figure 9 is a graph of the actual amplitude distribution of the pulses. On the other hand, if A_k is taken to be the same for all pulses, the curve of Figure 9 shows the values of $\dfrac{n_k^{1/2}}{k}$.

The parent population of pulses may thus be considered to be composed of pulses of various wave lengths, having the amplitudes and numbers as given by Figure 9. If a random sample of n of such pulses is taken and the sample is distributed at random over an interval of time an accelerogram will be formed. The average spectrum of a large number of

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[4] The random nature of earthquake ground motion was first pointed out in Characteristics of Strong-Motion Earthquakes, Bull. Seism. Soc. Amer. (1947) Vol.37. The spectra presented there were computed by means of a torsion pendulum that was later discovered to have approximately 0.01 of critical damping which was sufficient to eliminate the hump in the average spectrum curve and reduce it to a horizontal line. Details of spectrum calculation will be found in J. L. Alford, G. W. Housner and R. R. Martel, Spectrum Analysis of Strong-Motion Earthquakes, Office of Naval Research Report, Contract N6onr-244, California Institute of Technology, August 1951.
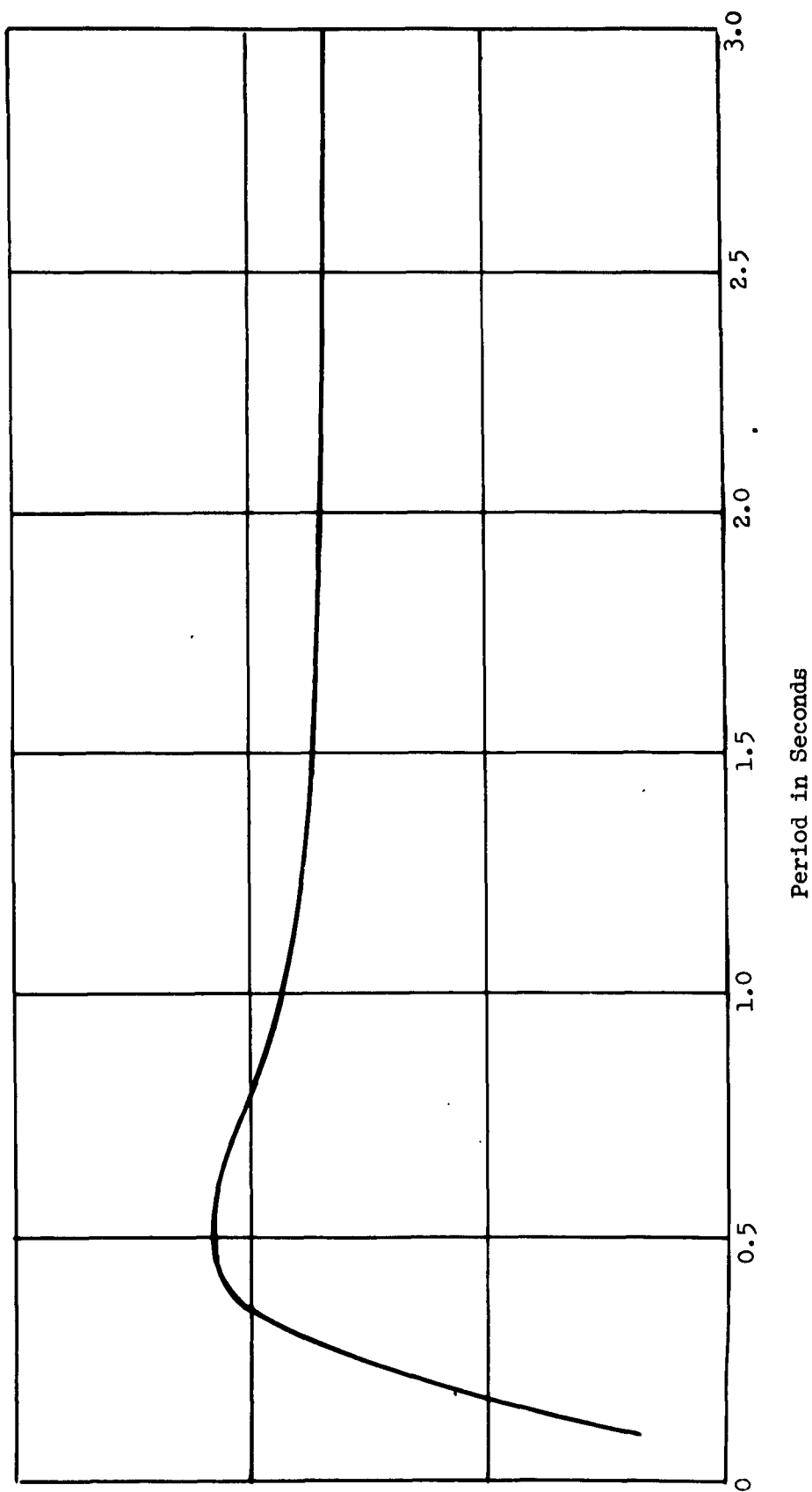
Period in Seconds

Fig. φ. Average Spectrum for Strong-Motion Earthquakes

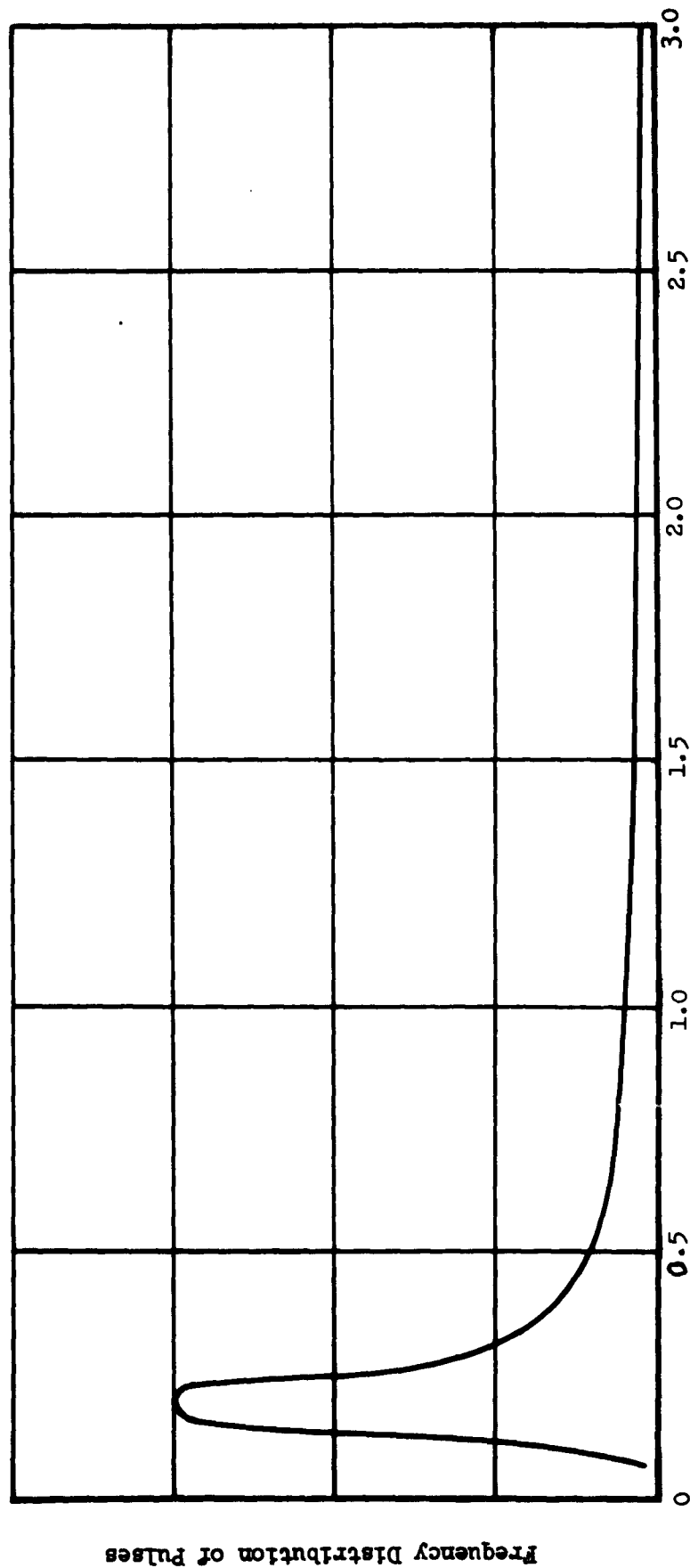Period of Pulse in Seconds

Fig. 8. Frequency Distribution of Pulses

Frequency Distribution of Pulses

accelerograms obtained in this way will approach the spectrum of Figure 8. A sample accelerogram obtained in this way will thus have the characteristics of actual recorded accelerograms in so far as these are random continuous functions.

A sample accelerogram was constructed by distributing 584 pulses over a 10 second interval, allotting 292 to each 5 second interval. To simplify the computations, the numbers and amplitudes of pulses were taken as shown in the following table so that the proper $\left(\dfrac{n^{1/2} A_k}{k}\right)$ was obtained, and these were distributed at random by means of a table of random numbers. This procedure will give a somewhat more uniform looking accelerogram than would be obtained if the pulses had been selected at random from the population and distributed at random over a 10 second period.

Wave Length	Number	Amplitude
0.1	200	5.0
0.2	100	11.0
0.3	66	4.0
0.4	50	1.5
0.5	40	1.0
0.6	33	0.58
0.7	28	0.37
0.8	25	0.25
0.9	22	0.17
1.0	20	0.12

The resulting accelerogram is shown in Figure 10. For comparison there is shown a portion of the Olympia, Washington accelerogram in Figure 11. It is seen that the two are very similar in appearance even to the extent that on the average they cross the axis the same number of times per second.

It should be noted that the preceding method of superposing the pulses actually corresponds to the time of strong motion only, that is, when very large numbers of pulses are arriving. The earlier and later portions of recorded accelerograms usually show many fewer pulses arriving.
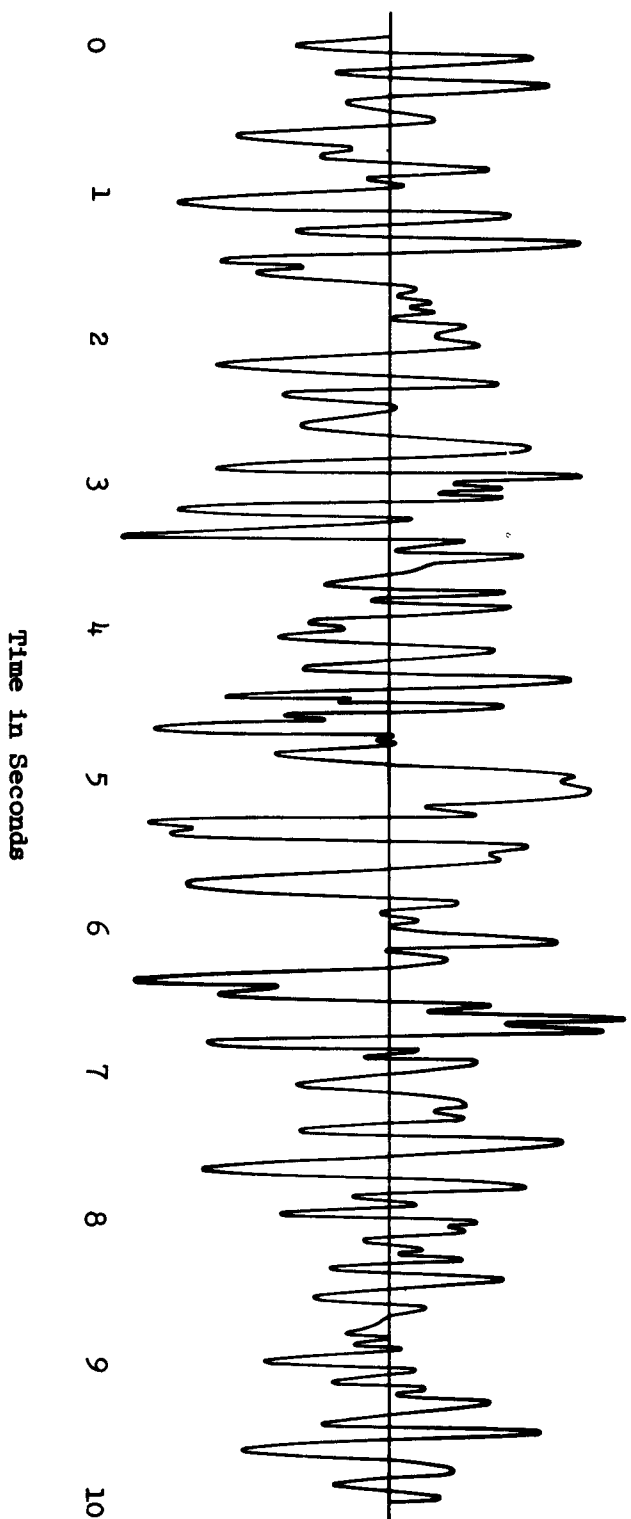
Time in Seconds

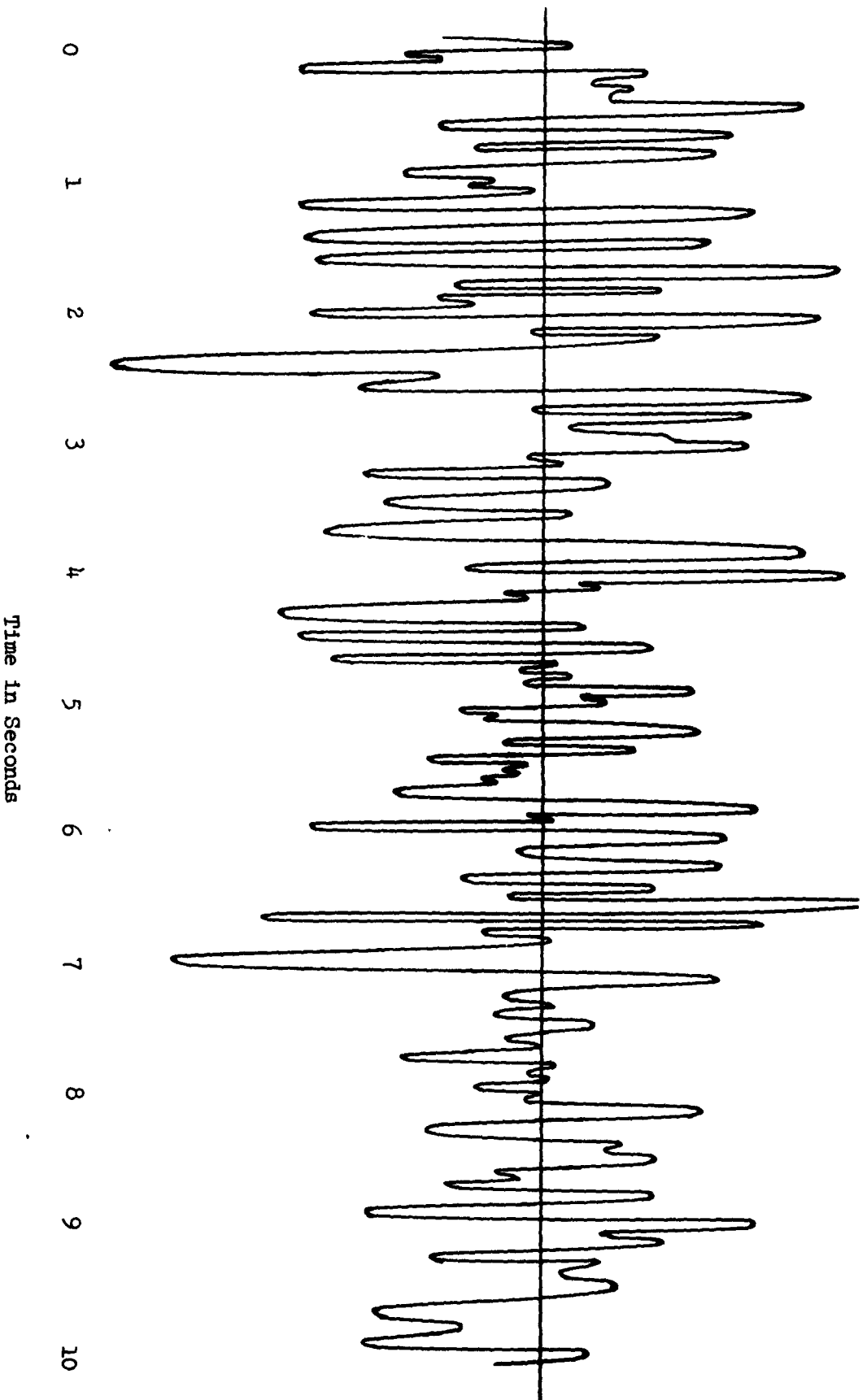Fig. 10. Accelerogram Formed by Random
Superposition of Pulses

Fig. 11. Portion of Olympia, Washington Accelerogram

Time in Seconds

MAXIMUM ACCELERATIONS

When an accelerogram is constructed with a set of pulses in the
foregoing manner it is found that, to a considerable degree, the pulses
cancel each other and that the maximum accelerations are the result of
fortuituous superpositions of pulses. If a second accelerogram is con-
structed using the same set of pulses it will differ from the first be-
cause of the random superposition of the pulses. However, if a large
number of accelerograms are constructed using the same set of pulses
they will have certain average properties, for example, there will be
an average maximum acceleration and the maximum accelerations of the
individual accelerograms will deviate from the average with a certain
statistical distribution. In what follows, the references to accelera-
tions are to the average properties.

When the average energy spectrum is calculated for a given dis-
tribution of pulses its ordinates, as shown by equation (12), are pro-
portional to (nA^2), where n is the number of pulses and A is the ampli-
tude. On the other hand, if accelerograms are constructed from the
pulses and the spectra computed for these, the average spectrum will
have ordinates proportional to the square of the maximum acceleration .
Therefore, the maximum acceleration α is proportional to the ampli-
tude of the pulses and to the square root of the number of pulses, that
is,

$$\alpha = K(n)^{\frac{1}{2}} A$$

where the number n is the pulse density at the point where the accel-
erations are measured, that is, the number of pulses per unit time.
If an increment of fault area dx dh is radiating pulses at a rate n per
unit area, the maximum acceleration at a specified point on the surface
of the gound will have a maximum acceleration

$$\alpha = K (n\ dx\ dh)^{\frac{1}{2}} A \tag{13}$$

If the point on the surface of the ground is sufficiently far from the
fault so that the radiation can be considered to come from a point source
the maximum acceleration recorded will be the cumulative acceleration

from the total area that is radiating, that is

$$\alpha^2 = K^2 \iint n A^2 \, dx \, dh \tag{14}$$

If the radiation is the same from all points on the fault area the maximum acceleration will be

$$\alpha = C \, A \left(N \right)^{\frac{1}{2}}$$

where N is the rate of pulse radiation of the entire fault area. N is directly proportional to the area, so the maximum acceleration α is proportional to the square root of the fault area. In terms of the magnitude

$$\alpha = C_1 \, e^M$$

This is essentially Richter's definition of magnitude, in terms of acceleration instead of displacement, which is thus a measure of the area of slip.

For points relatively close to the fault, it is not correct to assume point source radiation but the area of the fault must be taken into account. A qualitative investigation of this effect may be made as follows. Consider a fault of length 2ℓ and vertical dimension h_o, each point of which is radiating pulses according to equation (13). Furthermore, let the effect of the position of the point on the surface of the ground, where the accelerations are measured, relative to the point on the fault be described by the inverse square law with cosine correction as given by Gutenberg and Richter[5], that is

$$\alpha = K \, (n \, dx \, dh)^{\frac{1}{2}} \, A \, \frac{h}{x^2 + y^2 + h^2} \tag{16}$$

where x, y, h are the coordinates of the point on the surface of the ground with respect to the radiating point. The effect of the radiation from the fault area can be approximated by considering the radiation from a line of length 2ℓ at an equivalent depth h if the strength of the radiation is

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[5] B. Gutenberg and C. F. Richter, Earthquake Magnitude, Energy, Intensity and Acceleration, Bull. Seism. Soc. Amer., Vol.32, No.3, 1942.

taken to be proportional to h_o . If, then, the maximum acceleration is calculated for the line in accordance with equations (14) and (15) there is obtained

$$\alpha = \frac{\alpha_0 (h_o)^{\frac{1}{2}}}{2} \frac{h}{(h^2+y^2)^{3/4}} \left\{ \frac{(x+\ell)(h^2+y^2)^{\frac{1}{2}}}{(x+\ell)^2+h^2+y^2} - \frac{(x-\ell)(h^2+y^2)^{\frac{1}{2}}}{(x-\ell)^2+h^2+y^2} \right.$$

$$\left. + \tan^{-1} \frac{x+\ell}{\sqrt{h^2+y^2}} - \tan^{-1} \frac{x-\ell}{\sqrt{h^2+y^2}} \right\}^{\frac{1}{2}} \qquad (17)$$

where x, y, h are the coordinates of the point on the surface of the ground as measured in a coordinate system with origin at the center of the line, x and y being respectively parellel and perpendicular to the line and h being the vertical distance from the center of the line to the surface of the ground. Equation (17) describes the variation of seismicity over the surface of the ground. When $\ell \to$ o with ($\alpha_0 \ell^{\frac{1}{2}}$) constant, there is obtained for point source radiation

$$\alpha = (\alpha_0 (h_o \ell))^{\frac{1}{2}} \frac{h}{(x^2+y^2+h^2)} \qquad (18)$$

which agrees with equation (15).

According to equation (12), $\alpha = C_1 e^M$ and according to equation (4) the energy released is $E = C_2 e^{3M}$ so α is proportional to the cube root of the energy and equation (18) can be written

$$\alpha = C(E)^{\frac{1}{3}} \frac{h}{x^2+y^2+h^2} \qquad (18a)$$

The same equation is derived by Gutenberg and Richter[3] by using empirical equations based on observations.

According to equation (17), if $\frac{\ell}{\sqrt{h^2+y^2}}$ is smaller than approximately 0.20 the fault may be considered to be a point source but if the accelerations are measured closer to the fault than this, the effect of the dimensions of the fault must be taken into account. For example, consider a point directly above the center of the fault at x = 0, y = 0.

Equation (17) is then

$$\alpha = \alpha_o \left(\frac{h_o}{2h}\right)^{\frac{1}{2}} \left\{\frac{1}{\left(\frac{h}{\ell} + \frac{\ell}{h}\right)} + \tan^{-1}\frac{\ell}{h}\right\}^{\frac{1}{2}} \qquad . \quad (19)$$

When $\frac{\ell}{h}$ is less than 0.2 equation (19) can be written

$$\alpha = \alpha_o \left(\frac{h_o}{h}\right)^{\frac{1}{2}} \left(\frac{\ell}{h}\right)^{\frac{1}{2}}$$

and the radiation may be considered as originating from a point source. On the other hand, when ℓ becomes greater than this the effect of the increased length of fault upon the maximum accelerations rapidly decreases as shown in the table. It is seen that α calculated from equation (19) is virtually unaffected by increase in ℓ beyond ℓ/h = 1.2, so that an earthquake such as that of El Centro, 1940, for which ℓ was approximately 20 miles and the equivalent h perhaps 10 miles, the maximum accelerations in the vicinity of the center of the fault would not have been materially increased for an ℓ of 40 or more miles. Thus, as regards the effect of area of slip, the maximum accelerations associated with the El Centro shock are close to the maximum possible.

ℓ/h	$\frac{\alpha}{\alpha_o}\left(\frac{2h}{h_o}\right)^{\frac{1}{2}}$
0	0
.2	.62
.4	.85
.6	.99
.8	1.08
1.0	1.13
1.2	1.17
∞	1.25

SUMMARY

The dislocation theory considers that an earthquake fault is
formed by the superposition of a large number of incremental shear dis-
locations whose sudden release produces the earthquake. It is postu-
lated that during an earthquake the incremental dislocations are re-
leased in such a way that the average slip is proportional to the square
root of the area of slip, and that the probability of release of indivi-
dual incremental dislocations is such that the probability of a total
slip area A is inversely proportional to A. With these two postulates
a frequency distribution of earthquakes is derived that agrees with
observed data within the limits of accuracy of the data; the Richter
magnitude is shown to be essentially a logarithmic measure of the
average slip on a fault; and an expression is derived for the energy
released by an earthquake that agrees with that derived from considera-
tion of the energy carried in a wave train. Expressions are derived
also for the areas of slip during earthquakes, the maximum relative
slip, and the average annual, overall shearing distortion of the State
of California and these are in satisfactory agreement with observed
behavior. According to the theory, during an earthquake a large number
of incremental dislocations are released, each of which produces an
elemental acceleration pulse. An accelerogram is thus formed by the
superposition of a large number of elemental acceleration pulses ran-
dom in time. It is shown that this agrees with recorded accelerograms
and an accelerogram composed in this fashion is shown to have the
characteristics of actual recorded accelerograms. It is also shown
that the maximum ground accelerations in the vicinity of the center of
the fault, in so far as they are dependent upon the size of the slip
area, have essentially reached their upper limits for shocks with areas
of slip approximately equal to that associated with the El Centro, 1940
earthquake.

The fact that there is good agreement between the theory and
observations increases confidence that the relatively small number of

recorded strong ground motions are typical of the motions to be expected. For if shocks are generated by the release of dislocations, the ground motions must always be similar to the past recorded ground motions, that is, the energy spectra of the ground motion must be similar, for any radical departure would require a marked difference in the physical properties of the earth's crust.